

EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Solutions

Final Examination

September 2015

Question 1: Fixed Income Valuation and Analysis / Derivative Valuation and Analysis (64 points)

a) a1) Bond prices calculation: $P = \sum_{t=1}^{T} \frac{CF_{t}}{(1+R_{0,t})^{t}} = \frac{CF_{1}}{(1+R_{0,1})^{1}} + \frac{CF_{2}}{(1+R_{0,2})^{2}} + \dots + \frac{CF_{T}}{(1+R_{0,T})^{T}}$ Bond 1 => $P_{B1} = \frac{3\%}{(1+0.2\%)^{1}} + \frac{3\%}{(1+0.3503\%)^{2}} + \frac{3\%+100\%}{(1+0.5514\%)^{3}} = 107.288$ Bond 2 => $P_{B2} = \frac{6\%}{(1+0.2\%)^{1}} + \frac{6\%}{(1+0.3503\%)^{2}} + \frac{6\%+100\%}{(1+0.5514\%)^{3}} = 116.212$ YTM calculation: $P = \sum_{t=1}^{T} \frac{CF_{t}}{(1+YTM)^{t}} = \frac{CF_{1}}{(1+YTM)^{1}} + \frac{CF_{2}}{(1+YTM)^{2}} + \dots + \frac{CF_{T}}{(1+YTM)^{T}}$ YTM bond 1 =>107.288 = $\frac{3\%}{(1+YTM_{B1})^{1}} + \frac{3\%}{(1+YTM_{B1})^{2}} + \frac{3\%+100\%}{(1+YTM_{B1})^{3}}$ $\Rightarrow YTM_{B1} = 0.544\%$ YTM bond 2 =>116.212 = $\frac{6\%}{(1+YTM_{B2})^{1}} + \frac{6\%}{(1+YTM_{B2})^{2}} + \frac{6\%+100\%}{(1+YTM_{B2})^{2}}$

a2)

The yield to maturity of two bonds having the same maturity but different cash flows is not necessarily the same, even if the interest they pay is reinvested at the same rate.

Here, the bond with lower coupon has a higher YTM. This is because the YTM is a complex weighted average of the spot rates. The relative weight of the final cashflow [last coupon + redemption] is greater for a low coupon bond, and the IRS curve has a positive slope so the spot rates for longer maturities are higher.

The term structure is a "calculated" curve and deals with the relationship between zerocoupon yields (or spot rates) and time to maturity, whereas the yield curve is an "observed" curve and deals with yield to maturity and time to maturity.

The term structure of interest rates is the basis for the calculation of the present value of cash flows, such that the spot rates are expressed in a unique relationship between rates and the relative tenor, and also they are not contaminated by the effect of the reinvestment of coupons.

a3)

It would be possible to select the correct YTM using the concept of YTM as a complex average of spot rates. Being the choice among:

-Bond 1: 0.544% vs 0.559%; -Bond 2: 0.538% vs 0.556%;

0.559% (bond 1) and 0.556% (bond 2) are higher than the 3-year spot rate (0.5514%). This means that those values cannot be consistent with the concept of average of spot rates.

a4)

Lower coupon means a higher duration; so for the same expiry bond 1 has a higher duration.

b)

b1)

Default risk:

The risk that there will be a failure to pay interest or principal on the bond, or that there will be an equivalent failure on another bond issued by the same borrower. The concept is taken further in that most bonds include in their issuing circular details a cross default clause, which in effect states that any default on any of the issuer's bonds issued in the past or future will constitute a default on this particular bond. This means in effect that a default on one interest payment is a default on all of the issuer's debt (whether in the form of bonds or not: cross default clauses most often include all of the issuer's debt, whether in loan or bond form).

You can assess credit risk by analyzing the sector of the company and by making a ratio analysis.

The variables involved in its assessment are:

- Industry and Company considerations:

And in particular considering the economic cyclicality; growth prospects; investments in research and developments; competition; source of supply; degree of regulation.

- <u>Ratio analysis</u>:

The financial resources needed to service the debt issued can come from three sources: cash flow from operations; liquidation of some asset; another source of financing. In the long run, however, the ability to repay debt comes essentially from cash flows from operations. Consequently, bondholders need to know something about the cash flows that the issuer is likely to generate. The main source of information relies on the analysis of balance sheet and income statement accounts and is synthesized in the analysis of financial ratios that relate different items of income statement and balance sheet.

- Credit ratings:

The relative credit-risks of long-term bonds are assessed by various independent financial services firms which are known as the rating agencies. Their analysts analyze the various financial data such as fundamentals of the company, industry data and the macro-economic data to determine the possibility of default in interest and/or principal payments. Finally, based on these analyses, rating agencies assign a rating to the issuer which is published in various publications available to investors.

- Credit migration:

The usual measure of credit migration is known as a transition matrix. Based on the empirical evidence of a given credit agency's rating on a given bond being changed within a given period, a transition matrix measures the probability that a bond with a given rating at the start of the given period will have the same or a different rating at the end of the period.

- <u>Recovery rates:</u>

In the event of a default, the recovery rate is, as its name implies, the proportion of what is owed that is eventually repaid. It measures the likely absolute return in the event of default. It is also influenced by the management of credit event.

- Bankruptcy processes:

Even if the recovery rate is certain, the timing of that recovery has a crucial effect on the calculable expected return of a credit bond. In practice this dimension of credit is left to specialists and is not a notable feature of credit in the regular bond market.

b2)

Let S be the credit spread; the calculation is:

$$P = \sum_{j=1}^{n} \frac{CF}{\left(1 + R_{ZC}^{Tj} + S\right)^{Tj}} + \frac{100\%}{\left(1 + R_{ZC}^{Tn} + S\right)^{Tn}}$$

$$99.25\% = \frac{6.5\%}{\left(1 + 0.2\% + 5.777\%\right)^{1}} + \frac{6.5\%}{\left(1 + 0.3503\% + 5.777\%\right)^{2}} + \frac{6.5\%}{\left(1 + 0.5514\% + 5.777\%\right)^{3}}$$

$$+ \frac{6.5\%}{\left(1 + 0.7029\% + 5.777\%\right)^{4}} + \frac{6.5\% + 100\%}{\left(1 + 0.9580\% + 5.777\%\right)^{5}}$$

$$99.25\% \neq \frac{6.5\%}{\left(1 + 0.2\% + 4.505\%\right)^{1}} + \frac{6.5\%}{\left(1 + 0.3503\% + 4.505\%\right)^{2}} + \frac{6.5\%}{\left(1 + 0.5514\% + 4.505\%\right)^{3}}$$

$$+ \frac{6.5\%}{\left(1 + 0.7029\% + 4.505\%\right)^{4}} + \frac{6.5\% + 100\%}{\left(1 + 0.9580\% + 4.505\%\right)^{5}}$$

The credit spread at issue date is therefore equal to 577.7 basis points.

An S of 577.7bp should be used.

b3) Yield to call calculation at call date, March 1st 2018: 99.25% = $\frac{6.5\%}{(1+7.716\%)^1} + \frac{6.5\%}{(1+7.716\%)^2} + \frac{6.5\%+103\%}{(1+7.716\%)^3}$

Negative convexity: at low yield levels, the price/yield relationship for a callable bond will depart significantly from the price/yield relationship for the non-callable (bullet) equivalent bond. It is due to the price compression (there is a limited price appreciation as the yield declines).

b4)

The price of a similar security but without the call feature, i.e. a bullet bond, would be:

$$P_{\text{bullet}} = \frac{6.5\%}{\left(1+5.85\%\right)^{1}} + \frac{6.5\%}{\left(1+5.85\%\right)^{2}} + \frac{6.5\%}{\left(1+5.85\%\right)^{3}} + \frac{6.5\%}{\left(1+5.85\%\right)^{4}} + \frac{6.5\%+100\%}{\left(1+5.85\%\right)^{5}} = 102.75$$

Since a callable bond is a bullet bond plus a short call option we have: $P_{Callable bond} = P_{bullet} - Value of call option$

Value of call option = $P_{\text{bullet}} - P_{\text{Callable bond}} = 102.75 - 99.25 = 3.5$

call option price embedded in the bond equals 3.5%.

An increase in interest rate volatility implies an increase of the volatility of the bond prices, therefore an increase in the call price.

b5)
(i) Average life of the bond at the time of issue
Average life =
$$\sum_{t=1}^{n}$$
 Principle to be repaid (t) \cdot t
Averagelife = 20% \cdot 1 + 20% \cdot 2 + 20% \cdot 3 + 20% \cdot 4 + 20% \cdot 5 = 20% \cdot (15) = 3

(ii) Amortizing bond issue price:

$$P = \sum_{j=1}^{n} \frac{\text{Principle ro be repaid}(j) + CF(j)}{\left(1 + R_{ZC}^{T_j} + OAS\right)^{T_j}} \#$$

Cash flow CF(j) is calculated starting from cash flow CF(1) and reducing the following coupon by the same amortizing percentage.

Price calculation at credit spread = 590bp:

$$P = \frac{20\% + 6.5\%}{(1+0.2\% + 5.9\%)^{1}} + \frac{20\% + 5.2\%}{(1+0.3503\% + 5.9\%)^{2}} + \frac{20\% + 3.9\%}{(1+0.5514\% + 5.9\%)^{3}} + \frac{20\% + 2.6\%}{(1+0.7029\% + 5.9\%)^{4}} + \frac{20\% + 1.3\%}{(1+0.958\% + 5.9\%)^{5}}$$
= 99.90%

(iii) Amortizing bond price calculated at the 1st amortizing date (March 1st 2016):

Price calculation at credit spread = 590bp:

$$P = \frac{25\% + 6.5\%}{(1+0.5008\% + 5.9\%)^{1}} + \frac{25\% + 4.875\%}{(1+0.7276\% + 5.9\%)^{2}} + \frac{25\% + 3.25\%}{(1+0.8711\% + 5.9\%)^{3}} + \frac{25\% + 1.625\%}{(1+1.1484\% + 5.9\%)^{4}}$$
= 99.37%

Alternative price calculation method:

$$79.493\% = \frac{20\% + 5.2\%}{(1 + 0.5008\% + 5.9\%)^{1}} + \frac{20\% + 3.9\%}{(1 + 0.7276\% + 5.9\%)^{2}} + \frac{20\% + 2.6\%}{(1 + 0.8711\% + 5.9\%)^{3}} + \frac{20\% + 1.3\%}{(1 + 1.1484\% + 5.9\%)^{4}}$$

Therefore, $P = \frac{79.493\%}{80\%} = 99.37\%$

b6)

Considering a +5bp parallel shift of the IRS Spot curve, the bullet bond shows a higher interest rates sensitivity, because the full nominal redemption is concentrated in the 5th year, while the amortizing bond has an annual nominal redemption of 20%.

A numerical verification [not required in the question]:

New Price Calculation	BULLET BOND	OAS calculated a	at 590bp)					
Total Cash flows	-99,043	6,5	6,5	6,5	6,5	106,5		
Coupon payment dates	01/03/15	01/03/16	01/03/17	01/03/18	01/03/19	01/03/20		
Time to next cash flow		1	2	3	4	5		
IRS Spot curve		0,2000%	0,3503%	0,5514%	0,7029%	0,9580%		
OAS Spread		5,9000%	5,9000%	5,9000%	5,9000%	5,9000%		
Cash flows P.V.	98,744	6,12630	5,75776	5,38840	5,03312	76,43881		
Total Cash flows	-99,043	6,5	6,5	6,5	6,5	106,5		
Coupon payment dates	01/03/15	01/03/16	01/03/17	01/03/18	01/03/19	01/03/20		
Time to next cash flow		1	2	3	4	5		
IRS Spot curve+5bp shift		0,2500%	0,4003%	0,6014%	0,7529%	1,0080%		
OAS Spread		5,9000%	5,9000%	5,9000%	5,9000%	5,9000%		
Cash flows P.V.	98,540	6,12341	5,75234	5,38081	5,02369	76,26023	∆ Price	-0,20
Cash flows P.V.				5,38081	5,02369	76,26023	∆ Price	-0,20
	98,540 AMORTIZING BOND			5,38081	5,02369	76,26023	∆ Price	-0,20
Cash flows P.V.				5,38081	5,02369	76,26023 21,3	∆ Price	-0,20
Cash flows P.V. New Price Calculation	AMORTIZING BOND	OAS calculated a	at 590bp)	23,9		21,3	∆ Price	-0,20
Cash flows P.V. New Price Calculation Total Cash flows	AMORTIZING BOND -100,088	OAS calculated a	at 590bp) 25,2	23,9	22,6	21,3	∆ Price	-0,20
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates	AMORTIZING BOND -100,088	OAS calculated a	at 590bp) 25,2 01/03/17	23,9 01/03/18	22,6	21,3	Δ Price	-0,2
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow	AMORTIZING BOND -100,088	OAS calculated a 26,5 01/03/16 1	at 590bp) 25,2 01/03/17 2	23,9 01/03/18 3	22,6 01/03/19 4	21,3 01/03/20 5	Δ Price	-0,2
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow IRS Spot curve	AMORTIZING BOND -100,088	OAS calculated a 26,5 01/03/16 1 0,2000%	at 590bp) 25,2 01/03/17 2 0,3503%	23,9 01/03/18 3 0,5514%	22,6 01/03/19 4 0,7029%	21,3 01/03/20 5 0,9580% 5,9000%	Δ Price	-0,21
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow IRS Spot curve OAS Spread	AMORTIZING BOND -100,088 01/03/15	OAS calculated a 26,5 01/03/16 1 0,2000% 5,9000%	at 590bp) 25,2 01/03/17 2 0,3503% 5,9000%	23,9 01/03/18 3 0,5514% 5,9000%	22,6 01/03/19 4 0,7029% 5,9000%	21,3 01/03/20 5 0,9580% 5,9000%	Δ Price	-0,2
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow IRS Spot curve OAS Spread Cash flows P.V.	AMORTIZING BOND -100,088 01/03/15 99,899	OAS calculated a 26,5 01/03/16 1 0,2000% 5,9000% 24,97644	at 590bp) 25,2 01/03/17 2 0,3503% 5,9000% 22,32238	23,9 01/03/18 3 0,5514% 5,9000% 19,81271 23,9	22,6 01/03/19 4 0,7029% 5,9000% 17,49976	21,3 01/03/20 5 0,9580% 5,9000% 15,28776 21,3	Δ Price	-0,2
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow IRS Spot curve OAS Spread Cash flows P.V. Total Cash flows	AMORTIZING BOND -100,088 01/03/15 99,899 -100,088	OAS caculated a 26,5 01/03/16 1 0,2000% 5,9000% 24,97644 26,5	at 590bp) 25,2 01/03/17 2 0,3503% 5,9000% 22,32238 25,2	23,9 01/03/18 3 0,5514% 5,9000% 19,81271 23,9	22,6 01/03/19 4 0,7029% 5,9000% 17,49976 22,6	21,3 01/03/20 5 0,9580% 5,9000% 15,28776 21,3	Δ Price	-0,2
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow IRS Spot curve OAS Spread Cash flows P.V. Total Cash flows Coupon payment dates	AMORTIZING BOND -100,088 01/03/15 99,899 -100,088	OAS caculated a 26,5 01/03/16 1 0,2000% 5,9000% 24,97644 26,5	at 590bp) 25,2 01/03/17 2 0,3503% 5,9000% 22,32238 25,2	23,9 01/03/18 3 0,5514% 5,9000% 19,81271 23,9 01/03/18	22,6 01/03/19 4 0,7029% 5,9000% 17,49976 22,6	21,3 01/03/20 5 0,9580% 5,9000% 15,28776 21,3	Δ Price	-0,2
Cash flows P.V. New Price Calculation Total Cash flows Coupon payment dates Time to next cash flow IRS Spot curve OAS Spread Cash flows P.V. Total Cash flows Coupon payment dates Time to next cash flow	AMORTIZING BOND -100,088 01/03/15 99,899 -100,088	OAS caculated a 26,5 01/03/16 1 0,2000% 5,9000% 24,97644 26,5 01/03/16 1	at 590bp) 25,2 01/03/17 2 0,3503% 5,9000% 22,32238 25,2 01/03/17 2	23,9 01/03/18 3 0,5514% 5,9000% 19,81271 23,9 01/03/18 3	22,6 01/03/19 4 0,7029% 5,9000% 17,49976 22,6 01/03/19 4	21,3 01/03/20 5 0,9580% 5,9000% 15,28776 21,3 01/03/20 5	Δ Price	-0,2

c)

(i) Conversion value calculation:

Conversion value = Conversion ratio \cdot Share value in EUR = $10 \cdot 77.5 = EUR \ 775$

(ii) Straight bond value calculation:

$$89.47\% = \frac{3.5\%}{(1+6\%)^1} + \frac{3.5\%}{(1+6\%)^2} + \frac{3.5\%}{(1+6\%)^3} + \frac{3.5\%}{(1+6\%)^4} + \frac{3.5\%+100\%}{(1+6\%)^5}$$

Therefore, $\frac{89.47}{100} \cdot 1000 = \text{EUR 894.7}$

(iii) Convertible bond minimum value calculation: Min value = Max(775 EUR; 894.7 EUR) => EUR 894.7

Question 2: Derivative Valuation and Analysis

(38 points)

a) We have the relation $C = S \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2)$ $N(d_1) = \frac{C + K \cdot e^{-r \cdot \tau} \cdot N(d_2)}{S} = \frac{8.86 + 200 \cdot e^{-0.02 \cdot 0.25} \cdot 0.498}{200} = 0.5398$

b) Seller of 1000 calls, delta = 0.5398 / call Delta position: $-1000 \cdot 0.5398 = -539.8$ delta points.

To cancel the delta exposure, you have to create a +540 delta point position with equities (by definition the delta of an equity is 1.00).

So, you buy 540 stocks of MUNICH RE: $+540 \cdot 1 = +540$.

[with the given data: Seller of 1000 calls, delta = 0.5500 / call Delta position: $-1000 \cdot 0.5500 = -550$ delta points. To cancel the delta exposure, you have to create a +550 delta point position with equities (by definition the delta of an equity is 1.00). So, you buy 550 stocks of MUNICH RE: $+550 \cdot 1 = +550$.]

c)
c1)

$$d_{1} = \frac{\ln(S/K) + (r + \sigma^{2}/2)\tau}{\sigma\sqrt{\tau}} = \frac{\ln(202/200) + (0.02 + 0.21^{2}/2) \cdot 0.25}{0.21 \cdot \sqrt{0.25}} = 0.1949$$

$$N(d_{1}) = N(0.1949) = N(0.19) + 0.5 \cdot (N(0.20) - N(0.19))$$

$$= 0.5753 + 0.5 \cdot (0.5793 - 0.5753) = 0.5773$$
Delta = N(d_{1}) = 0.5773

 $d_2 = d_1 - \sigma \cdot \sqrt{\tau} = 0.1949 - 0.21 \cdot \sqrt{0.25} = 0.0899$ N(d₂) = 0.5358

$$C = S \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2) = 202 \cdot 0.5773 - 200 \cdot e^{-0.02 \cdot 0.25} \cdot 0.5358 = 9.98$$

c2)

Value of 1000 call options sold at 8.86: $-1000 \cdot (9.98 - 8.86) = -1120$ EUR Value of 540 stocks bought at 200: $+540 \cdot (202 - 200) = +1080$ EUR Loss of 40 EUR

[With the given data: Value of 1000 options sold at 8.86: $-1000 \cdot (10 - 8.86) = -1140$ EUR Value of 550 stocks bought at 200: $+550 \cdot (202 - 200) = +1100$ EUR Loss of 40 EUR] This loss can be due to:

- convexity. Delta neutral hedging is not perfect (should use delta/gamma neutral to be more perfectly hedged)
- Rounding effects of calculations
- Change in volatility

The passing of time is not a right answer since the price move occurred only one minute after the hedge.

d)

d1)

Since the delta neutral is not a perfect hedge because of the convexity of the option price curve, we need to introduce a second derivative (like convexity / sensitivity for bonds). It allows the hedge to be more efficient for small and medium size variations of the underlying asset.

Delta neutral is built by trading the underlying asset, whose delta is (by definition) equal to 1.00. But the underlying asset does not have any gamma: thus, we need a 'gamma bearing' product to hedge the gamma of an option.

We use another option (on the same underlying asset) to neutralize the gamma. We then measure a new delta (the second option modified the global delta position), and we neutralize it with the underlying asset.

d2) Seller of 1000 call 200, delta 0.575, gamma 0.0369 Delta position: $-1000 \cdot 0.575 = -575$ delta points Gamma position: $-1000 \cdot 0.0369 = -36.9$ gamma points

(i) Neutralizing the gamma with puts gamma 0.0341 (finding +36.9 gamma points) $\frac{+36.9}{0.0341} = +1082$ So, we have to buy 1082 put options

(ii) Calculating the new delta Initial delta position: -575 (call 200) Additional delta: $+1082 \cdot (-0.3928) = -425$ (put 198) TOTAL DELTA: -1000.00

(iii) Neutralizing residual deltaBUY 1000 MUNICH RE stocks(number required: as 550 stocks were already bought at question b), the market maker would have to buy 450 more stocks)

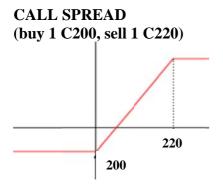
_	Unit delta	Unit gamma	Quantity	DELTA	GAMMA
Call 200	0.5750	0.0369	-1000	-575	-36.9
Put 198	-0.3928	0.0341	+1082	-425	+36.9
Stocks	1.0000	0.0000	+1000	+1000	+0.0
				0.0	0.0

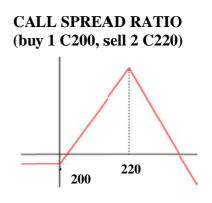
e)

To achieve this goal, the investor can build a CALL SPREAD, by selling another call "out of the money" (on the same underlying asset, MUNICH RE).

For example, he could sell 1 call 220 (for each call 200 bought): price received by the investor would be around 2.4 EUR (calculation not required!), and thus would reduce the total cost to \sim 6.46 EUR (-27%).

He could even sell more calls: 2 C220 for each C200 bought for example, creating a CALL SPREAD RATIO. The price of the strategy would be more than halved compare to the call only, but the investor would take a risk if MUNICH RE stock price really rallies.





Question 3: Derivatives / Derivatives in Portfolio Management

(32 points)

a)

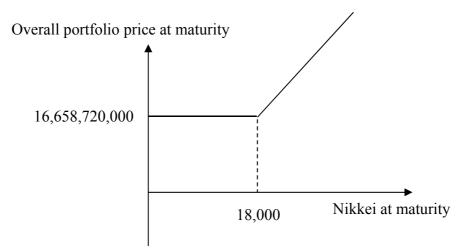
The value of the portfolio is 1 million times the Nikkei (= 20 billion yen / 20,000 yen). If at maturity the value of the Nikkei is less than the 18,000 yen strike price of the put option, your option position must earn enough profit to offset the 1 million times the loss on the Nikkei in order for the overall price of the portfolio to remain constant. You therefore purchase 1,000 trading units of put options (1 million / 1,000).

 $N_{p} = \frac{S_{0}}{I_{0} \cdot k} = \frac{20,000,000,000}{20,000 \cdot 1,000} = 1,000$

The current purchasing price of 1,000 trading units of put options with a strike price of 18,000 yen is $N_p \cdot k \cdot P = 1,000 \cdot 1,000 \cdot 1,328 = 1,328,000,000$ yen. This is borrowed at the risk-free rate and repaid at maturity. The amount of the repayment is therefore: $N_p \cdot k \cdot P \cdot (1 + r\tau) = 1,328,000,000 \cdot (1 + 4\% \cdot 0.25) = 1,341,280,000$ yen.

The floor will be 18 billion yen less this amount, so the value of the floor that you achieve will be 16,658.72 million yen.

b)



c)

It is possible to increase the floor achieved at the end of 3 months by selling call options with a strike price higher than 18,000 and investing the proceeds at the risk-free rate. This raises the floor by sacrificing the profit earned if there is an increase in the Nikkei. Compared to b), the overall value of the portfolio will be lower if there is a rise in the Nikkei.

d)

The delta of a put option with strike price 18,000 yen is -0.296. Meanwhile, the futures price is equivalent to the no-arbitrage price, so the futures price is [as stated in the question, "dividends can be ignored for the sake of simplicity"]:

 $I_0 \cdot (1 + r \cdot \tau) = 20,000 \cdot (1 + 4\% \cdot 0.25) = 20,000 \cdot 1.01 = 20,200$ yen.

This results in a delta of 1.01 for the futures. Therefore, a dynamic hedge of 1 trading unit put options requires futures per option of $N_F = \frac{\Delta_P}{\Delta_F} = \frac{-0.296}{1.01} = -0.293$ trading units (a short position of 0.293 trading units).

What is actually required is 1,000 times this, so the futures position to be taken at the current point of time for a dynamic hedge is 1,000 times 0.293, or a short position of 293 trading units.

e)

Pros:

- The options market has less liquidity than the futures market, so the purchase of options could have large market impact, which may cause prices to surge higher.
- Using futures to synthesize a dynamic hedge may be cheaper than purchasing the options themselves.

Cons:

- Conversely, trading cannot in actual practice be performed consecutively and there are trading costs that are incurred. This can result in deviation from the synthesis of a dynamic hedge, so that the more you trade the higher the ultimate cost.
- Another problem with dynamic hedges in addition to this is the stochastic fluctuation of the underlying asset volatility in light of the option gamma.

f)

Under the no-arbitrage condition, put/call parity is a relationship between the price C of a European-style put option and the price P of a European-style call option with the same time to maturity τ , the same underlying asset and strike price K, their common underlying asset price S and the risk-free rate r:

$$C - P = S - \frac{K}{1 + r \cdot \tau}$$

Applying this to a put and call with strike prices of 20,000 yen, put/call parity is not achieved, as shown below.

$$C - P = 2,414 - 2,274 = 140$$

$$S - \frac{K}{1 + r \cdot \tau} = 20,000 - \frac{20,000}{1 + 4\% \cdot 0.25} = 198.02$$

Therefore, there is an arbitrage opportunity. The arbitrage can be set up in the following way:

- Purchase 1 trading unit of call options
- Sell 1 trading unit of put options
- Short position of 1 trading unit of Nikkei futures

- Lend
$$\frac{20,000}{1+4\% \cdot 0.25} \cdot 1000 = 198020$$
 yen (3 months) at the risk-free rate

At the current point in time, this position is

$$1000 \cdot \left(-C + P + S - \frac{K}{1 + r \cdot \tau}\right)$$

= 1000 \cdot \left(- 2414 + 2274 + 20,000 - \frac{20,000}{1 + 4\% \cdot 0.25} \right)
= 1000 \cdot \left(-140 + 198.02 \right)
= 58,020 yen

This is the profit that will be obtained; the payoff at maturity (3 months) is offset, resulting in 0.

Question 4: Portfolio Management

a)

Ex-ante TE: Forecast of the standard deviation (or variance) of the active return for a given time period.

Ex-post TE: Retrospective measurement of the standard deviation (or variance) of the active return for a given time period.

Since the ex-ante TE has to be lower than 4%, an active performance of -8% to 8% implies, plus and minus 2 standard deviations from the mean (0), and we know this probability to be 95.44%. The probability is approximately 95%.

b) Given the covariance matrix:

 $C = \begin{bmatrix} 0.20 & 0.03 \\ 0.03 & 0.05 \end{bmatrix}$

And the active weights: $W_A^P = \begin{bmatrix} 0.2 & -0.2 \end{bmatrix}$

Tracking Error Variance is calculated as:

 $W_{A}^{P'} \cdot C \cdot W_{A}^{P} = \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix} \begin{bmatrix} 0.20 & 0.03 \\ 0.03 & 0.05 \end{bmatrix} \begin{bmatrix} 0.2 & -0.2 \end{bmatrix} = 0.0076$

Tracking Error Standard deviation: $\sqrt{0.0076} = 0.0872$

Or alternatively we can calculate as: TEVar = $0.2^2 \cdot 0.2 + (-0.2)^2 \cdot 0.05 + 2 \cdot 0.2 \cdot (-0.2) \cdot 0.03 = 0.076$ TESD = $\sqrt{0.0076} = 0.0872$

Remark: In the current case, the tracking error standard deviation is larger than the restriction set by the client of 4%.

c)

The calculation can be done using the definition of the ex-ante information ratio, which is the quotient of the expected active return and the (expected) active risk:

 $I\widetilde{R}_{A}^{P,B} = \frac{R_{A}^{P,B}}{T\widetilde{E}_{A}^{P,B}}$

Solve for the expected return: $R_A^{P,B} = I\widetilde{R}_A^{P,B} \cdot T\widetilde{E}_A^{P,B} = 0.8 \cdot 4\% = 3.2\%$ The expected active return is 3.2%. d)

The formula states that a manager's value-added (Information Ratio) is a function of his prediction skill (Information Coefficient) and the number of opportunities (N). Following this law an active manager has two possibilities to achieve higher information ratios. He can increase his skills or the number of bets. In practice, it is hard to increase forecasting skills, consequently a manager should bet as often as possible, if he thinks he has an edge.

$$IR = IC \cdot \sqrt{N} \Longrightarrow IC = \frac{IR}{\sqrt{N}} = \frac{0.8}{\sqrt{4}} = 0.4$$

An information coefficient of 0.4 is a big number for quarterly market forecasts. Of course it is not impossible to obtain such results over a longer time period; however, it is an optimistic anticipation.

e)

- The TAA aims to outperform a strategic benchmark by actively over/underweighting individual asset classes.
- These allocation decisions are based on some forecasts and/or projections.
- The time horizon of a typical TAA is on the order of weeks or months.
- A TAA can be implemented via cash and/or derivative transactions.

f)

The portfolio insurance gives the investor the ability to limit the downside risk while allowing some participation in the upside market. It is a path dependent self-financing dynamic strategy that maintains the portfolio's risk exposure a constant multiple of the excess of wealth over a floor, up to a borrowing limit. It involves the exchange of two assets on the financial market: the riskless asset usually treasury bills or liquid money market instruments and the risky asset. The percentage allocated to each depends on the "cushion" value, defined as (current portfolio value – floor value), and a multiplier coefficient, where a higher number denotes a more aggressive strategy.

In general portfolio insurance strategies turn out to be not very expedient in a low-rate environment. In the case of CPPI, the cushion size is determined by the rate level. Low rates imply a small cushion and accordingly require a larger investment multiple to achieve a target exposure of the risky asset. This means greater risk-taking and tends to exacerbate the 'cash lock-in' issue.

Question 5: Portfolio Management

a)

The formula for the beta of a portfolio is:

$$\boldsymbol{\beta}_{p} = \sum_{i=1}^{N} \boldsymbol{w}_{i} \cdot \boldsymbol{\beta}_{i} = \boldsymbol{w}_{L} \cdot \boldsymbol{\beta}_{L} + \boldsymbol{w}_{S} \cdot \boldsymbol{\beta}_{S}$$

where, WL = weight of long positions and WS = weight of short positions

Substituting, we have:

$$0 = w_L \cdot 1.5 - w_S \cdot 1.2 \implies \frac{w_L}{w_S} = \frac{1.2}{1.5} = 0.8$$

[Note: The result could also be expressed as WS/WL = (1.5) / (1.2) = 1.25]

b)

The reasoning behind the long-short, zero-beta strategy is twofold:

- i) The zero-beta approach is expected to avoid market risk. The approach will not eliminate all risk, however, since idiosyncratic risk will still remain.
- ii) The long-short approach is expected to capitalize on the ability of the security analyst to differentiate between undervalued stocks (the long portion of the portfolio) and overvalued stocks (the short portion of the portfolio). The approach is expected to produce a return from both portions of the portfolio.